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A STUDYFROM THE OPERATOR OF EVOLUTION AND THE DESCRIPTIONS OF HEISENBERG, OF SCHRODINGER AND "OF INTERACTION"AND ITS VERIFICATIONS IN SERIES THROUGH POWER SERIES EXPANSIONS, CONCOMITANTLY WITH THE INTEGRATION OF H(T) IN AND EXPONENTIAL, FOR EXAMPLE, FUNCTION THE RIEMANN ZETA

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SUMMARY

This article provides an outline and a brief analysis of what will be done.in the study of the topic From the article. From Mechanics Quantum equations have the function ofEvolution Operator, which in the description ofHeisenberg in the Avatars of Mechanics Quantum mechanics is incorporated into the Equation of Motion. The first phase is to analyze the Exponential Function, as an example,It is comparedthe Function Riemannian Zeta, wherethere are severalpossibilities with the tools ofMathematic, such asExpansionsfrom the SeriesBy Taylorin Mathematics. After the first phase, an analysis and comparison is made of what could be done with the Riemann Zeta function, as an example, inEuler's treatment and conclusion regarding the Zeta function, through its relation to prime numbers, leads to the conclusion that the Riemann Zeta function and its related prime numbers have the same relationship with the hydrogen atom and the Schrödinger equation. As an example, the Zeta function is discussed. Riemann's attempt to associate with the Evolution Operator, along with theEquation of Motion. Remember thatThe Exponential Factorfrom the Operator of Evolution, beyond the exponential terthe Factorof Integral that Associates the Hamiltonianwhich can and should be analyzedAccording to Conceptual Physics in the Future.

KEYWORDS: Descriptionby Heisenberg, SeriesTaylor's Law and Rieman's Zeta Function.

INTRODUCTION

The description ofThe evolution over time of the observable properties of a quantum system appears embodied in variousdifferent forms, two ofwhich were adopted respectively by Heisenberg andputSchrodingeralready on the occasion of the double invention of the theorythere are threeQuarter-century. The purpose of this section is to present these two descriptions.and a third one, also of considerable usecurrent, emphasizingat the same time its common core, which is an operator unitarydepending on the time period in effect, inIn the final analysis, the temporal evolution of the quantum system. The final subsection also deals with an important relationship between the time scales involved in the evolution of a given system.system state and degreeof the indeterminacy of energy in this state, this degree of indeterminacy being understood in the sense used in the discussion of uncertainty relations.[1].

Theoretical Frameworkand Discussion

2.3.1 The Evolution Operator and the Descriptions of Heisenberg, Schrodinger, and "Interaction"[1].

As indicated in section 2.1, based on the discussion of the equation of motion postulated in the context of matrix mechanics, in Heisenberg's description.The temporal evolution of Hermitian operators representing the variables (observables) of a quantum system is defined by an operator.also, Hermitian H (the operatorHamiltonian of the physical system considered)through the equation of motion(2.23)[1].

$$i\hbar \frac{dg(t)}{dt} = [g(t), h]$$

which, taking into account the commutation relation postulated for canonically conjugate coordinates and momenta, leads to equations of motion for these variables that maintain a complete formal analogy with Hamilton's canonical equations of classical mechanics.

Assuming thatH doesn't dependexplicitly from time, the equation of motion (2.23), supplemented by the initial condition that g(t=t0)=g, admits the formal solution.

$$g(t)=e^{\frac{i}{\hbar}H(t-t_0)}ge^{-\frac{i}{\hbar}H(t-t_0)} \equiv U^{\dagger}(t,t_0)gU(t,t_0)$$

which revealsexplicitly the fact that temporal evolution is effectively governed by the evolution operator

$$U(t,t_0) \equiv e^{-\frac{i}{\hbar}H(t-t_0)}$$

To theThe descriptions by Heisenberg and Schroedinger do not exhaust the alternatives offormulation forthedynamicsquantum. Also fromFrequently used formulations are "intermediate" formulations, in which the role of dynamic agents is of a certain...distributed among the variablesdynamicsand the state vectors. The situationA typical instance where these descriptions are used is one in which the Hamiltonian operator, not explicitly dependent on time, is written as the sum of two parts thatThey also do not explicitly depend on time.

$$H=H_0 + H_1$$

This allows you to introduce, in addition to the operatorof evolution defined as in (2.25), an operator ofevolution is also unitary butassociated only with H0:

$$U(t,t_0) \equiv e^{-\frac{i}{\hbar}H(t-t_0)}$$

In this casefrom a time-dependent Hamiltonian H(t), an evolution operator U(t,t0) can be defined by trivially generalizing the differential equation (2.26) to

$$i\hbar\frac{U(t)}{dt} = H(t)U(t,t_0),$$

Keeping the initial condition unchanged(2.31)

$$U(t,t_0) = \hat{1}$$

Equation (2.31) can also be written underthe form of the integral equation, which incorporatesthe initial condition(2.32)

$$U(t,t_0) = \hat{1} - \frac{i}{\hbar}\int_{t_0}^{t} dt'\, H(t')U(t',t_0).$$

…[1]

An idea that one can reach, or an idea that one can go to.As an example, a brief explanation of whatIt can or could be done. Knowing that an example can only be an example, remembering thatthis example diversifiesfor all types of series and expansions in series. And that at a certain point, an analysis is being carried out ofA specific function, such as Euler's and Riemann's, the Zeta function, was explored. At a certain point in scientific research, the pattern of prime numbers in the Riemann function was established in the spectrum of atoms, such as the hydrogen atom. This article provides a brief outline of this idea, serving as an introduction to research and analysis, as well as a possible future application.can be done with others, by trial and error, through analysis, induction, intuition, by reducible forms and by proofs of absurdity.

Euler Product[2]

For $s > 1$ theRiemann zeta functionis given by

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \qquad (1)$$

$$= \prod_{k=1}^{\infty} \frac{1}{1-\frac{1}{p_k^s}}, \qquad (2)$$

where p_k is the k thprime. This is Euler's product (Whittaker and Watson 1990), called by Havil (2003, p. 61) the "all-important formula" and by Derbyshire (2004, pp. 104-106) the "golden key."

This can be provided by expanding the product, writing each term as ageometric series, expanding, multiplying, and rearranging terms,

$$\begin{aligned}
\prod_{k=1}^{\infty} \frac{1}{1-\frac{1}{p_k^s}} &= \frac{1}{1-\frac{1}{p_1^s}} \frac{1}{1-\frac{1}{p_2^s}} \frac{1}{1-\frac{1}{p_3^s}} \cdots \\
&= \left[\sum_{k=0}^{\infty}\left(\frac{1}{p_1^s}\right)^k\right]\left[\sum_{k=0}^{\infty}\left(\frac{1}{p_2^s}\right)^k\right]\left[\sum_{k=0}^{\infty}\left(\frac{1}{p_3^s}\right)^k\right]\cdots \\
&= \left(1+\frac{1}{p_1^s}+\frac{1}{p_1^{2s}}+\frac{1}{p_1^{3s}}+\ldots\right)\left(1+\frac{1}{p_2^s}+\frac{1}{p_2^{2s}}+\frac{1}{p_2^{3s}}+\ldots\right)\cdots \\
&= 1+\sum_{1\le i}\frac{1}{p_i^s}+\sum_{1\le i\le j}\frac{1}{p_i^s p_j^s}+\sum_{1\le i\le j\le k}\frac{1}{p_i^s p_j^s p_k^s}+\ldots \\
&= 1+\frac{1}{2^s}+\frac{1}{3^s}+\frac{1}{4^s}+\frac{1}{5^s}+\ldots \\
&= \sum_{n=1}^{\infty}\frac{1}{n^s} \\
&= \zeta(s).
\end{aligned} \tag{3}$$

Here, the rearrangement leading to equation (1) follows from thefundamental theorem of arithmeticsince each product ofprime powersappears inexactly one denominatorand eachpositive integerequalsexactly oneproduct of prime powers.

This product is related to theMöbius function $\mu(n)$via

$$\frac{1}{\zeta(s)} = \sum_{n=1}^{\infty}\frac{\mu(n)}{n^s}, \tag{4}$$

which can be seen by expanding the product to obtain

$$\frac{1}{\zeta(s)} = \prod_{k=1}^{\infty}\left(1-\frac{1}{p_k^s}\right) \tag{5}$$

$$= \left(1-\frac{1}{p_1^s}\right)\left(1-\frac{1}{p_2^s}\right)\left(1-\frac{1}{p_3^s}\right)\cdots \tag{6}$$

$$= 1-\left(\frac{1}{p_1^s}+\frac{1}{p_2^s}+\frac{1}{p_3^s}+\ldots\right)+\left(\frac{1}{p_1^s p_2^s}+\ldots+\frac{1}{p_1^s p_3^s}+\frac{1}{p_2^s p_3^s}+\ldots\right)-\ldots \tag{7}$$

$$= 1-\sum_{0<i}\frac{1}{p_i^s}+\sum_{0<i<j}\frac{1}{p_i^s p_j^s}-\sum_{0<i<j<k}\frac{1}{p_i^s p_j^s p_k^s}+\ldots \tag{8}$$

$$= \sum_{n=1}^{\infty}\frac{\mu(n)}{n^s}. \tag{9}$$

$\zeta(1) = \infty$,

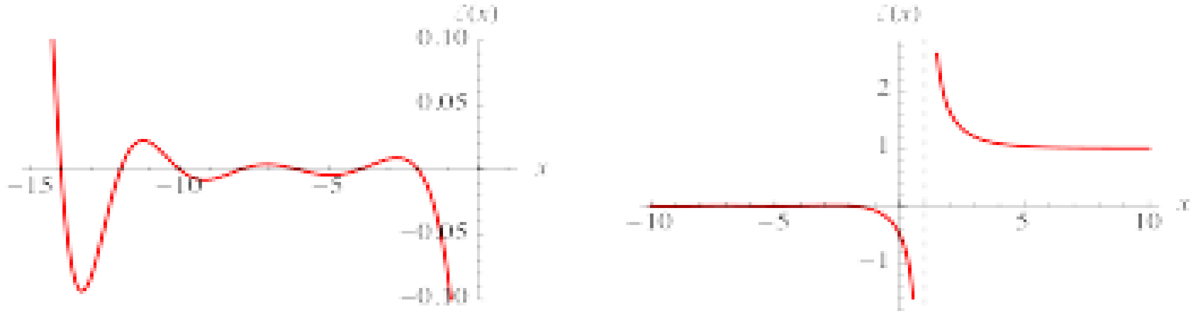
But the finite product exists, giving

$$P(n) = \prod_{k=1}^{n} \frac{1}{1 - \frac{1}{p_k}}. \qquad (10)$$

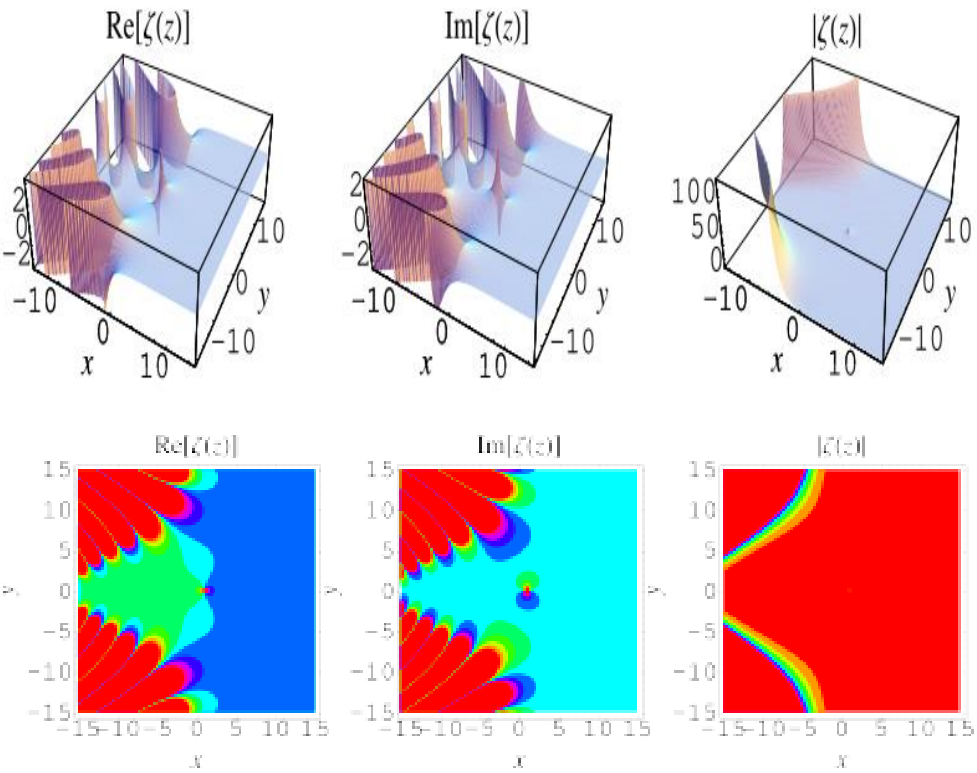
For upper limits $n = 0$, 1, 2, ..., the products are given by 1, 2, 3, 15/4, 35/8, 77/16, 1001/192, 17017/3072,... (OEISA060753andA038110). Premultiplying by $1/\ln p_n$ and letting $n \to \infty$ gives a beautiful result known as theMertens theorem.

The Euler product appears briefly in a pan of John Nash's (played by Russell Crowe) blackboard scribblings in Ron Howard's 2001 filmA Beautiful Mind.[2]

Riemann Zeta Function.[3]



The Riemann zeta function is an extremely importantspecial functionof mathematics and physics that arises in definite integration and is intimately related with very deep results surrounding theprime number theorem. While many of the properties of this function have been investigated, there remain important fundamental conjectures (most notably theRiemann hypothesis) that remain unprovided for this day. The Riemann zeta function is denoted $\zeta(s)$ and is plotted above (using two different scales) along the real axis.



In general, $\zeta(s)$ is defined over the complex plane for one complex variable, which is conventionally denoted s (instead of the usual z) in deference to the notation used by Riemann in his 1859 paper that founded the study of this function (Riemann 1859).

$\zeta(s)$ is implemented in theWolfram Languageto theZeta[s]

The plot above shows the "ridges" of $|\zeta(x+iy)|$ for $0<x<1$ and $1<y<100$. The fact that the ridges appear to decrease monotonically for $0 \leq x \leq 1/2$ is not a coincidence since it turns out that monotonic decrease implies theRiemann hypothesis(Zvengrowski and Saidak 2003; Borwein and Bailey 2003, pp. 95-96).

Remember that,

$$i\hbar \frac{dg(t)}{dt} = [g(t), h]$$

AND

$$U(t, t_0) \equiv e^{-\frac{i}{\hbar} H(t-t_0)}$$

Sendo that the naive formal solution of equation (2.31), possibly written as

$$U(t, t_0) \rightarrow e^{-\frac{i}{\hbar}\int_{t_0}^{t} H(t)dt} (?)$$

Thewhich would in fact be the correct solutionin the case whereIf H(t) were a numerical function, it is in reality excessively naive in this case. The solution duly takes into account the non-commutative nature of the Hamiltonian at different times, but it can be thought of, as in the example above, in terms of a succession of infinitesimal unitary transformations, which advance time in intervals dt during the time interval. The coefficient of H(t) is negligible.

$$U(t + dt, t) = \hat{1} - \frac{i}{\hbar}H(t)dt + \vartheta(dt^2)$$

The unitarity (up to order dt) of this infinitesimal transformation follows directly from the hermeticity of H(t), and the unitarity of the finite transformation, seen as a succession of these infinitesimal transformations, follows from the fact that the product of unitary operators is itself a unitary operator.

The intention of the analysis is as follows, as the function ZETA OF RIEMMAN EH DADAPOR,

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \qquad (1)$$

$$= \prod_{k=1}^{\infty} \frac{1}{1 - \frac{1}{p_k^s}}, \qquad (2)$$

And the exponential function is given by, TAYLOR'S SERIAL EXPANSION[4], The "series expansion of Taylor" means using the Taylor series to represent the exponential function, as an infinite sum of terms derived from its derivatives at a single point, often. The expansion for is a fundamental example, calculated as:

Input interpretation

series	e^x

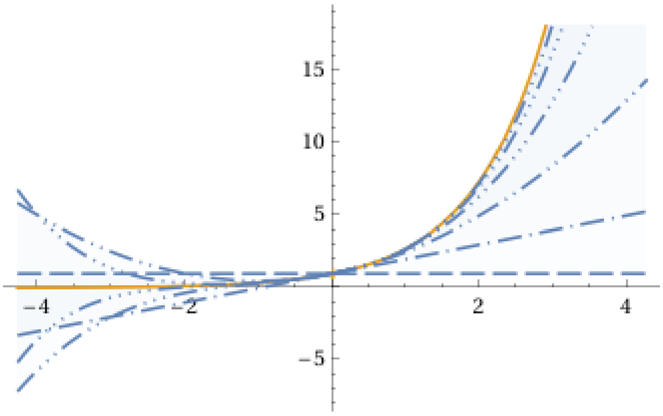
Series expansion at x=0

$$1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + O(x^6)$$

(Taylor series)

(converges everywhere)

Approximations about x=0 up to order 5



(order n approximation shown with n dots)

Series representation

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$e^x = \sum_{k=-\infty}^{\infty} I_k(x)$$

$$e^x = \sum_{k=0}^{\infty} \frac{x^{-1+2k}(2k+x)}{(2k)!}$$

$$e^x = \sum_{k=0}^{\infty} \frac{x^{2k}(1+2k+x)}{(1+2k)!}$$

Function approximation: AI uses Taylor series to approximate complex functions, such as , with simpler polynomial functions, which can be easier and faster for a computer to calculate. Algorithm building: These approximations are used in various AI algorithms to perform calculations, as shown in the C programming example where the first few terms of the expansion are used to approximate.

Error estimation: The Taylor series provides a way to estimate the error in these approximations, which is critical for building reliable models.

Simplified example: For a small value of, the Taylor approximation is highly accurate. For example, the first three terms can be a good estimate forwhen it is very close to 0.

The analysis will take place cwith the proof, that in 1737, the connection between the zeta function andprime numbers was discovered by Euler, who proved the identity

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p \text{ prime}} \frac{1}{1-p^{-s}},$$

where, by definition, the left hand side isζ(s) and theinfinite producton the right hand side extends over all prime numbersp(such expressions are calledEuler products):

$$\prod_{p \text{ prime}} \frac{1}{1-p^{-s}} = \frac{1}{1-2^{-s}} \cdot \frac{1}{1-3^{-s}} \cdot \frac{1}{1-5^{-s}} \cdot \frac{1}{1-7^{-s}} \cdot \frac{1}{1-11^{-s}} \cdots \frac{1}{1-p^{-s}} \cdots$$

Both sides of the Euler product formula converge for Re(s) > 1. Theproof of Euler's identityuses only the formula for thegeometric seriesand thefundamental theorem of arithmeticSince theharmonic series, obtained whens= 1, diverges, Euler's formula (which becomes Πp(p/p-1)) implies that there areinfinitely many primes.[5]Since the logarithm ofp/(p-1) is approximately 1/p, the formula can also be used to prove the stronger result that the sum of the reciprocals of the primes is infinite. On the other hand, combining that with theSieve of Eratosthenesshows that the density of the set of primes within the set of positive integers is zero.

The Euler product formula can be used to calculate theasymptomatic probabilitythatsrandomly selected integers within a bound are set-wisecoprime. Intuitively, the probability that any single number is divisible by a prime (or any integer)pis 1/pTherefore, the probability thatsnumbers are all divisible by this prime is 1/ps, and the probability that at least one of them isnotis 1 − 1/ps. Now, for distinct primes, these divisibility events are mutually independent because the candidate divisors are coprime (a number is divisible by coprime divisorsnandm if and only ifit is divisible bynm, an event which occurs with probability 1/(nm)). Thus the asymptotic probability thatsnumbers are coprime is given by a product over all primes,[5]

$$\prod_{p \text{ prime}} \left(1 - \frac{1}{p^s}\right) = \left(\prod_{p \text{ prime}} \frac{1}{1-p^{-s}}\right)^{-1} = \frac{1}{\zeta(s)}.$$

The analysis conditions must be applied to the equation. from the operator,

$$U(t,t_0) \to e^{-\frac{i}{\hbar}\int_{t_0}^{t} H(t)dt} (?)$$

The Analysisconceptual will beapplied to the conceptsofRiemann zeta function provides a profound link to the distribution of prime numbers via the Euler product formula and Riemann's explicit formulas. There is also a highly speculative, but actively researched, connection between the non-trivial zeros of the zeta function and the energy levels of quantum systems, but this does not involve the standard hydrogen atommodel. The term "electrum" is likely a misinterpretation of a mathematical or physical term, as it has no standard place in this context.

Prime Numbers and the Riemann Zeta Function

Euler Product Formula: For complex numbers with a real part greater than 1, the Riemann zeta function, can be expressed as an infinite product over all prime numbers. This fundamental identity reveals the intrinsic connection between the function and the "atoms of mathematics" (prime numbers).

Prime Number Distribution: The location of the non-trivial zeros of the zeta function is intimately linked to the accuracy of formulas that approximate the distribution of prime numbers among integers.

Riemann Hypothesis: The famous conjecture states that all non-trivial zeros of the zeta function lie on the "critical line" (where the real part of the complex number is 1/2). If proven true, it would provide the best possible estimate for the error term in the Prime Number Theorem, offering unparalleled.

Led insight into the seemingly random pattern of primes.

Quantum Mechanics and the Hydrogen Atom

Quantum Chaos: While there is no direct, provides link between the Riemann zeta function zeros and the exact energy levels of the standard hydrogen atom(which is anintegrablesystem), there is a compelling, unproven conjecture (the Hilbert-Pólya conjecture) that the zeros might correspond to the energy levels (eigenvalues) of some hypotheticalself-adjoint operatoror aquantum chaotic system.

Physics Applications: The zeta function does appear in various areas of physics, such as in quantum field theory calculations (eg, the Casimir effect) and statistical mechanics.

Hydrogen Spectrum: The energy levels of the hydrogen atom can be calculated using the simple Rydberg formula), which relates to basic quantum mechanics and the Schrödinger equation, but not directly to the distribution of the zeta function's zeros in a one-to-one manner.

Clarification

The term "electrum" is likely either a typo for "electron" or refers to an ancient gold alloy, neither of which has a standard, direct mathematical or physical relationship with the Riemann zeta function or prime numbers. In summary, the concepts connect via the mathematical properties of the zeta function related to prime distribution, and the theoretical speculation linking its zeros to quantum system energy levels, although not the simple, well-understood hydrogen atom.

CONCLUSION

Therefore, from now onThe analysis will be conducted, and this article will present it openly, over the next few months. It should be noted that this article was only a brief overview of the ideas and concepts.that are related in the analysis,Through the calculations that are performed as mentioned, such as the Riemann Zeta Function, which has a complex construction approach throughout the history of Mathematics and Physics, as outlined in the book The MusicPrime Numbers by Marcus du Sautoy. Do you still remember that?Reference andfrom the Explanation of the book Quantum Mechanics by David J. Griffiths[7] in reference to the Harmonic Oscillator and the Schrodinger Equation.

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